Inventory Management & Model Theory

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Modeling Definitions

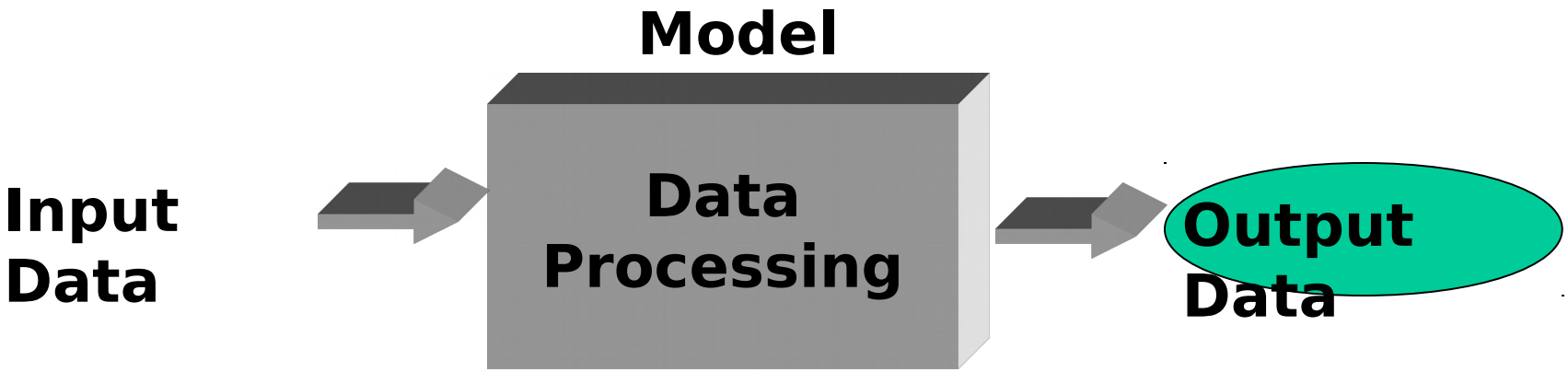
Model: An abstraction/representation of reality

- **Purpose is for prediction**
- **Develop understanding about real world process**

Data: Representation of facts, concepts or instructions in a formalized manner

- **Suitable for communications & interpretation**
- **Processed by human or automated means**

Modeling



Modeling Information

- **The output of the model can only be as good as its input**

- **The collection of accurate input data is therefore critical**

- **Sensitivity Analysis: Varying questionable input data over a range of values to assess its impact on the output data**

Model Types

Iconic: Using physical replica of the actual item example: Scaled down prototype

Analog: Using continuous variable data for abstraction of real world phenomena
example: Slide rules before calculators invented

Digital: Using discrete representation of data for the abstraction of real world phenomena
example: Calculator

Hybrid: Using continuous variable data & discrete data for abstraction of a real world phenomena example: Digital

Digital Model Types

Simulation: The representation of certain features of behavior of a physical or abstract system by the behavior of another system

- **Processes are essentially sequential**
- **Decisions are based on predetermined rules programmed into an automated evaluation procedure**

Analytical: The mathematical representation of certain features or behavior of a physical or abstract system

- **Processes are essentially calculated utilizing**

Inventory System Modeled Costs

- **Carrying Costs**
- **Shortage Costs**
- **Replenishment Costs**

Carrying Costs

- **Investment Cost: Money tied up in inventory not invested elsewhere**
- **Obsolescence**
  - **Technological**
  - **Over-forecasting of requirements**
  - **Deterioration**
- **Pilferage**
- **Taxes**
- **Insurance**
- **Warehousing**
- **Handing**

Shortage Costs

- **Overtime Cost**
- **Special clerical and administrative cost**
- **Loss of Specific sales } Loss of present sales**
- **Loss of goodwill } Loss of future sales**
- **Loss of customers } Loss of future sales**
- **Loss of end item usage**

Replenishment Costs

Ordering Cost:

- Clerical and administrative costs
- Transportation costs
- Handling costs

Setup Costs:

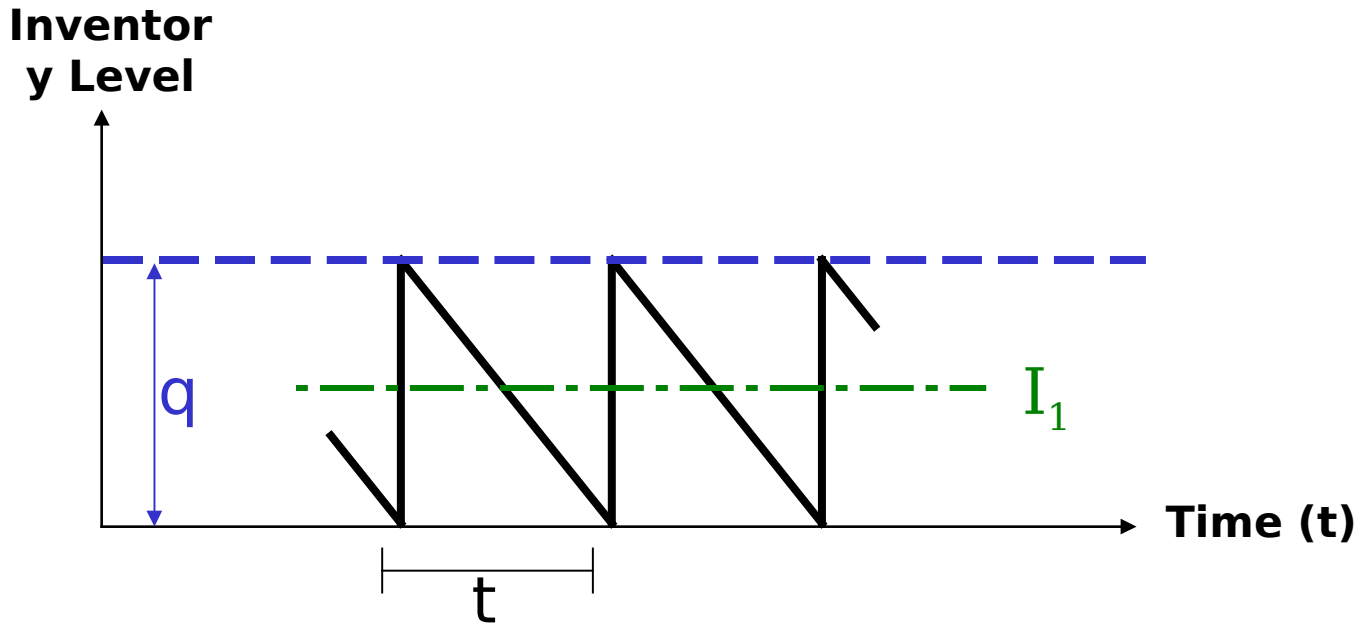
- Labor setup costs
- Cost of materials used during setup testing
- Cost of time during which production cannot take place due to this setup

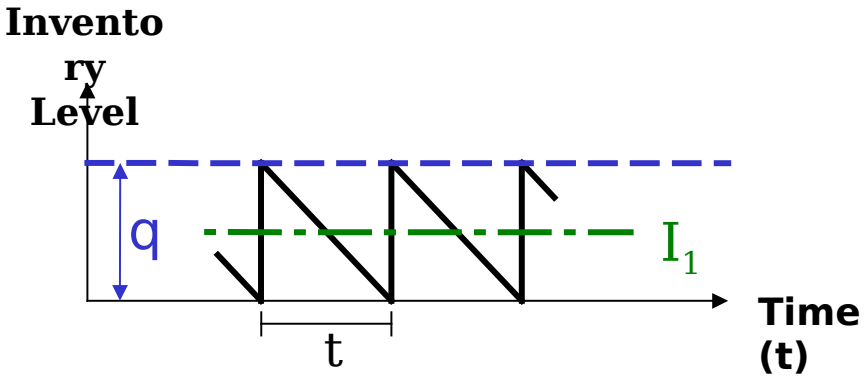
Procurement Demand Rate

- **Procurement Demand Rate Does Not Include Demands for Repair**
  - **Repair Costs Less Than Replenishment Buys Causing Repairs to be Pursued Before Purchasing Items**
  - **Applies Forecasted Demand Rate of Replenishment Buys for Best Model Input**

- **Procurement Demands**
  - **Demand Rate associated with Throwaway Items**
  - **Certain Repairable Items Demands:**
    - **Item Not Returned by User or Field for Higher Level Repair**
    - **Item Washed Out Because Repair is Not Economical**
  - **If Demand Rate Data Includes Repairs, apply Unserviceable Return Rate and Washout Rate Factors to Estimate Replenishment Demand Rate**

The Basic Inventory Model (Lot Size System)





- **Demand rate is r quantity per unit time**
- **Replenishment size is the lot size q**
- **t is the scheduling period**
- **Replenishment rate per unit time is infinite**
- **Replenishments are made whenever the inventory reaches the prescribed zero level**
- **Replenishment lead time is zero**
- **I_1 the average amount carried in inventory**

$$\mathbf{c = c_1 \cdot I_1 + \frac{c_3}{t}}$$

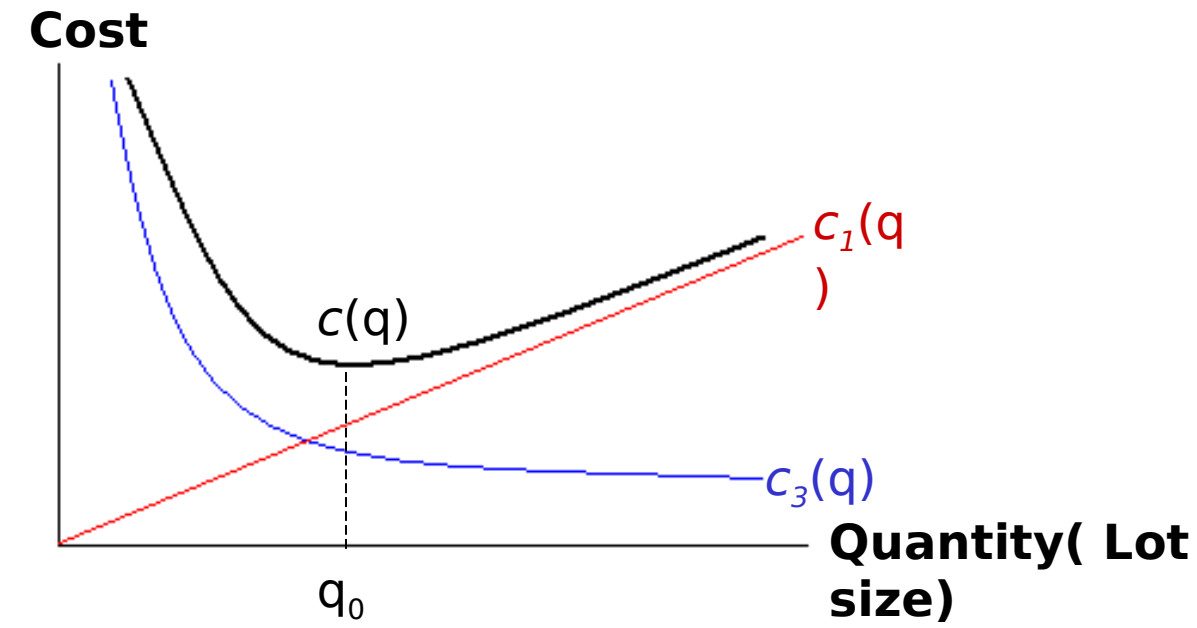
Where:

- C is the total cost per unit time
- C_1 is the unit carrying cost per unit time $\left[\frac{\$}{Q \cdot T}\right]$
- C_3 is the replenishment cost [$]

Cost vs. Quantity

$$c = c_1 \cdot I_1 + \frac{c_3}{t} \qquad I_1 = \frac{q}{2} \qquad t = \frac{q}{r}$$

$$\therefore c(q) = \frac{c_1 \cdot q}{2} + \frac{c_3 \cdot r}{q}$$



Economic Order Quantity (Optimal Lot Size)

Economic Order Quantity

By differentiating *c*(q) and setting the equation equal to zero, a minimum cost lot size can be determined

$$\frac{dc(q)}{dq} = \frac{c_1}{2} - \frac{c_3 r}{q_o^2} = 0$$

$$\frac{c_1}{2} = \frac{c_3 r}{q_o^2} \qquad c_1 q_o^2 = 2 c_3 r \qquad q_o^2 = \frac{2 r c_3}{c_1}$$

$$\boxed{q_o = \sqrt{\frac{2 r c_3}{c_1}}}$$

Economic Order Quantity

$$q_o = \sqrt{\frac{2rc_3}{c_1}}$$

Note: $c_1 = f \cdot p$

Where:

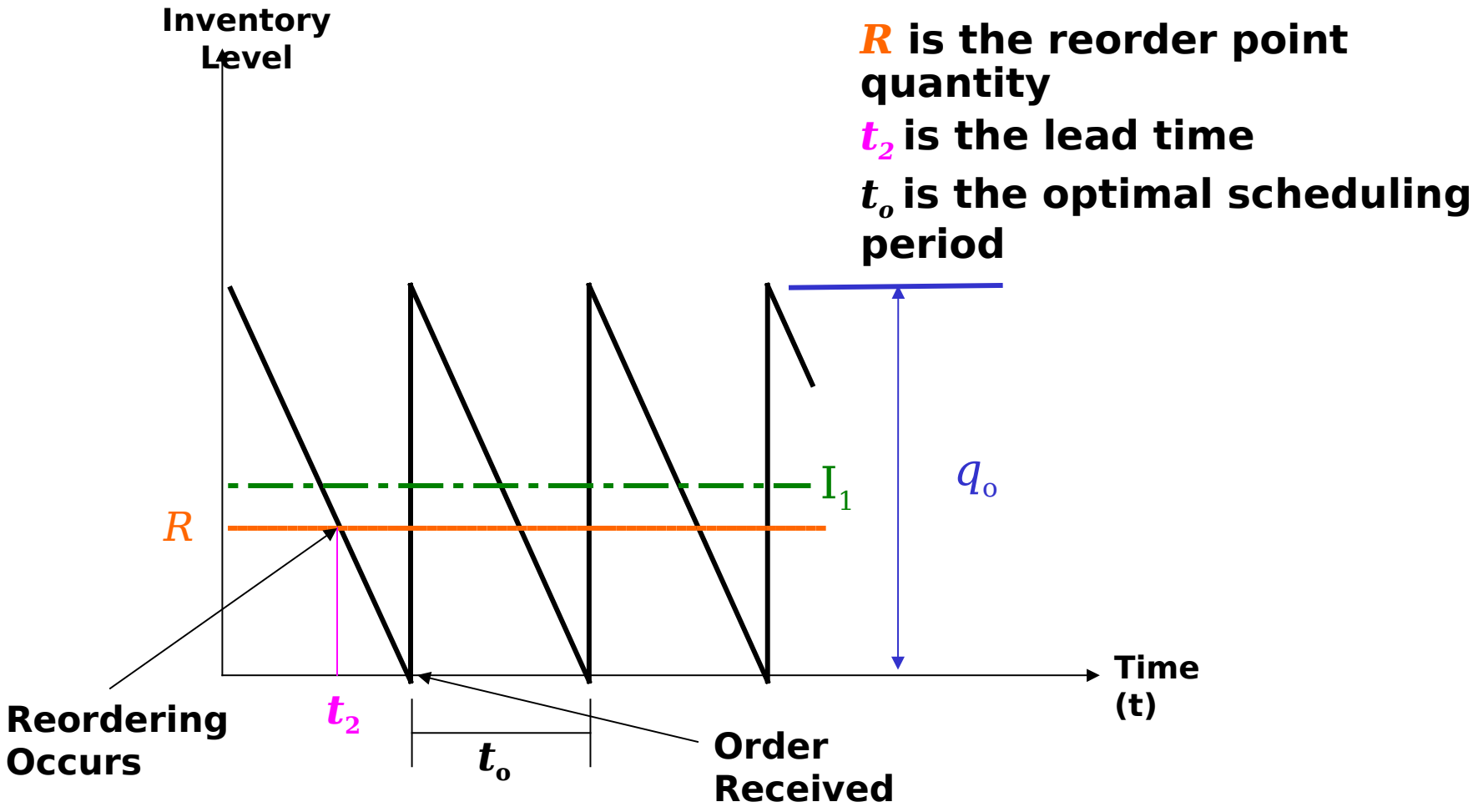
- ***f*** **is the carrying cost as a percentage of the unit price**
- ***p*** **is the unit price of the item in inventory**

$$\therefore q_o = \sqrt{\frac{2rc_3}{f \cdot p}}$$

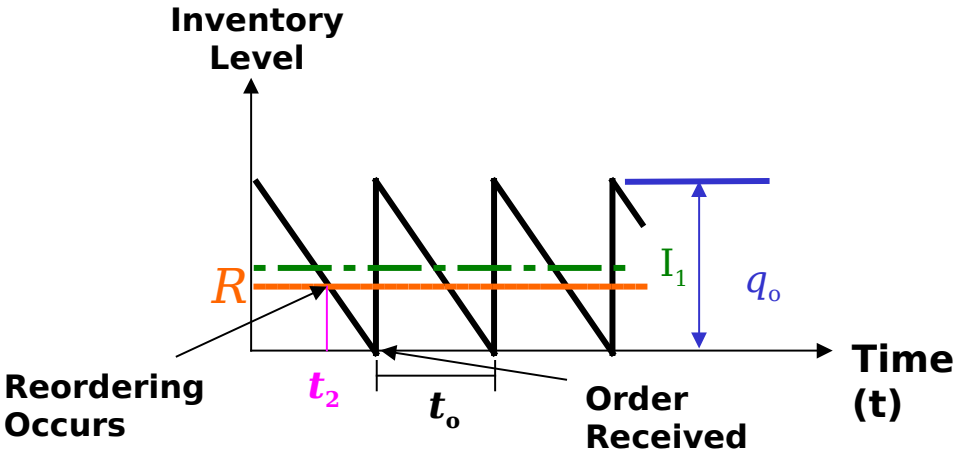
CCSS C-E Holding Cost Factors

- **Storage Cost – 1%**
- **Loss or Pilferage – 2%**
- **Investment Opportunity or Discount Rate – 7%**
  - **For Government, should use Net Discount Rate**
    - **Cost to Pay Government Debt minus Inflation Rate**
- **Obsolescence Rate**
  - **27.3% for year 1**
  - **6.9% for years 2 – 4**
  - **7.9% for years 4 – 12**
  - **9.8% for years 12 and beyond**
- **Disposal Cost (End of Life Application Only) – 2%**

Lot Size System Model with Replenishment Lead Time



Reorder Point Quantity



The reorder point quantity is the established level of inventory requiring order placement for the economic order quantity lot size

$$R = r \cdot t_2$$

Example

Suppose an inventory control problem has the following specifications for a particular item:

- **Demand rate: 25 units per week or 25 x 52 = 1300 units per year**
- **Unit price = $5**
- **Carrying cost factor = 20% per year**
- **Replenishment cost = $40**
- **Lead time = 4 weeks**

Economic Order Quantity:

$$q_o = \sqrt{\frac{2rc_s}{f \cdot p}} = \sqrt{\frac{2(1300)(40)}{(.2)(5)}} = 322.49 \approx 322 \text{ unit}$$

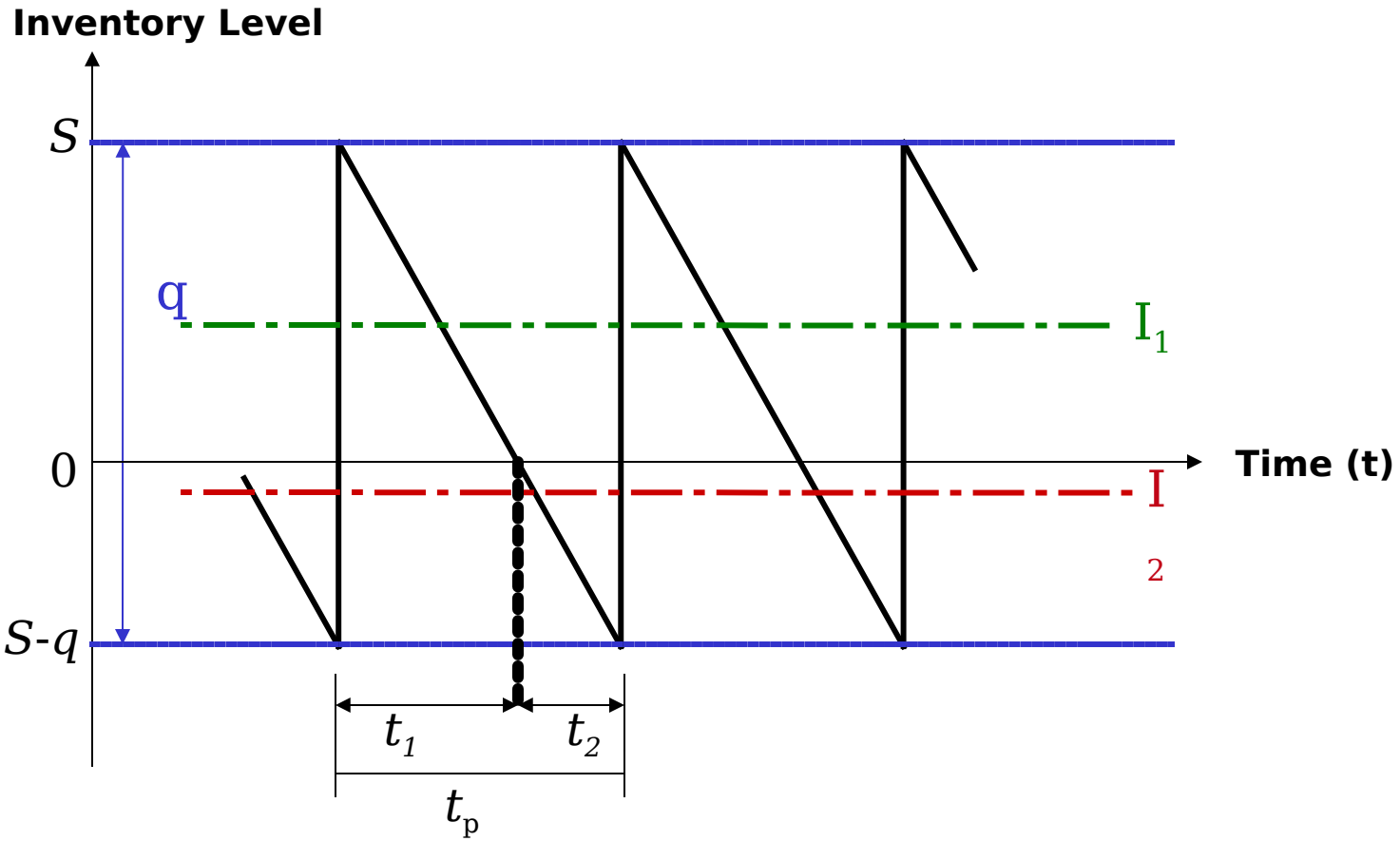
Reorder Point Quantity:

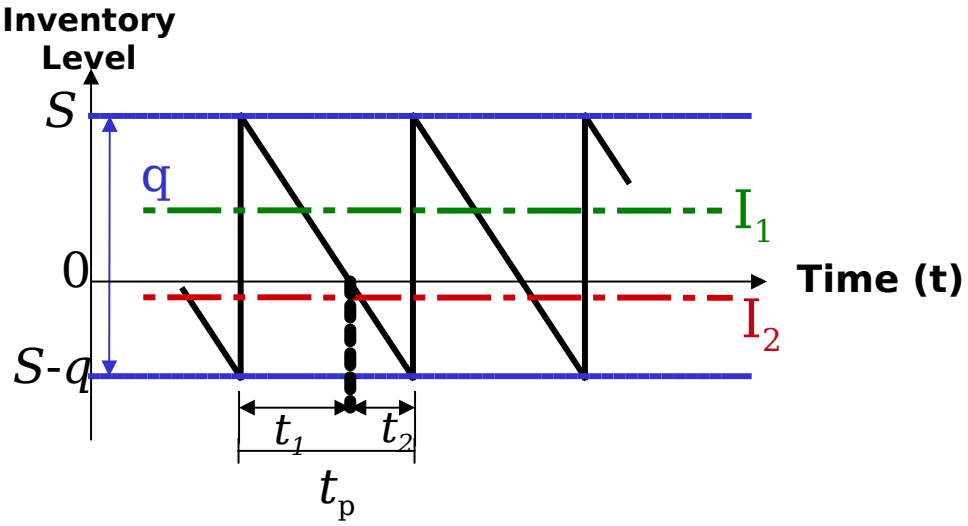
$$R = r \cdot t_2 = 25 \times 4 = 100 \text{ unit}$$

∴ An order for 322 units should be placed when the current inventory falls to a 4 week supply of 100 units.

Orders should be placed 1300 / 322 = 4.04 times per year

Order Level Lot Size System Model





- **Demand rate is *r* (quantity per unit time)**
- **Replenishment size is the lot size *q***
- **Replenishment rate per unit time is infinite**
- **Replenishment lead time is zero**
- **I_1 is the average amount carried in inventory**
- **t_p is the scheduling period**
- ***S* is the order level**
- **Replenishments are made whenever *q*-*S* backorders are reached**
- **I_2 is the average shortage amount**

$$c = c_1 I_1 + c_2 I_2 + \frac{c_3}{t_p}$$

Where:

- **c is the total cost per unit time**
- **c_1 is the unit carrying cost per unit time** $\left[\frac{\$}{Q \cdot T}\right]$
- **c_2 is the unit shortage cost per unit time** $\left[\frac{\$}{Q \cdot T}\right]$
- **c_3 is the replenishment cost [$]**

Note:

$$\frac{t_1}{t_p} = \frac{S}{q} \quad \& \quad \frac{t_2}{t_p} = \frac{q - S}{q} \quad \& \quad t_p = \frac{q}{r}$$

$$I_1 = \frac{S}{2} \cdot \frac{t_1}{t_p} + 0 \cdot \frac{t_2}{t_p} = \frac{S}{2} \cdot \frac{S}{q} = \frac{S^2}{2q}$$

$$I_2 = 0 \cdot \frac{t_1}{t_p} + \frac{q - S}{2} \cdot \frac{t_2}{t_p} = \frac{q - S}{2} \cdot \frac{q - S}{q} = \frac{(q - S)^2}{2q}$$

$$\therefore c(s,q) = \frac{c_1 S^2}{2q} + \frac{c_2 (q - S)^2}{2q} + \frac{c_3 r}{q}$$

By taking the partial derivative with respect to S, a minimum cost order level can be determined in terms of a minimum cost lot size.

$$\frac{\partial c}{\partial S} = \frac{c_1 S_o}{q_o} - \frac{c_2 (q_o - S_o)}{q_o} = 0$$

$$S_o = q_o \cdot \frac{c_2}{c_1 + c_2}$$

Reorder Point Quantity:

$$R = -(q_o - S_o) = -q_o + q_o \frac{c_2}{c_1 + c_2} = \frac{-c_1 q_o + c_2 q_o - c_2 q_o}{c_1 + c_2}$$

$$R = -q_o \cdot \frac{c_1}{c_1 + c_2}$$

By taking the partial derivative with respect to q, the minimum lot cost lot size can be determined.

$$\frac{\partial C}{\partial q} = -\frac{c_1 S_o^2}{2q_o^2} + \frac{c_2(q_o - S_o)}{q_o} - \frac{c_2(q_o - S_o)^2}{2q_o^2} - \frac{c_3 \cdot r}{q_o^2} = 0$$

$$-c_1 q_o^2 \left[\frac{c_2}{c_1 + c_2}\right]^2 + 2c_2 q_o^2 \left[\frac{c_2}{c_1 + c_2}\right] - c_2 q_o^2 \left[\frac{c_2}{c_1 + c_2}\right]^2 - 2c_3 r = 0$$

$$\frac{2c_1 c_2 q_o^2}{c_1 + c_2} - \frac{(c_1 + c_2) c_1 c_2 q_o^2}{(c_1 + c_2)^2} - 2c_3 r = 0$$

$$q_o = \sqrt{\frac{2rc_3}{c_1} \cdot \frac{(c_1 + c_2)}{c_2}}$$

$$S_o = q_o \cdot \frac{c_2}{c_1 + c_2} = \sqrt{\frac{2rc_3}{c_1} \cdot \frac{c_2}{c_1 + c_2}}$$

Reorder Point Quantity without replenishment lead time:

$$R = -q_o \cdot \frac{c_1}{c_1 + c_2} = -\sqrt{\frac{2rc_3}{c_2} \cdot \frac{c_1 + c_2}{c_1}} \cdot \left[\frac{c_1}{c_1 + c_2} \right]$$

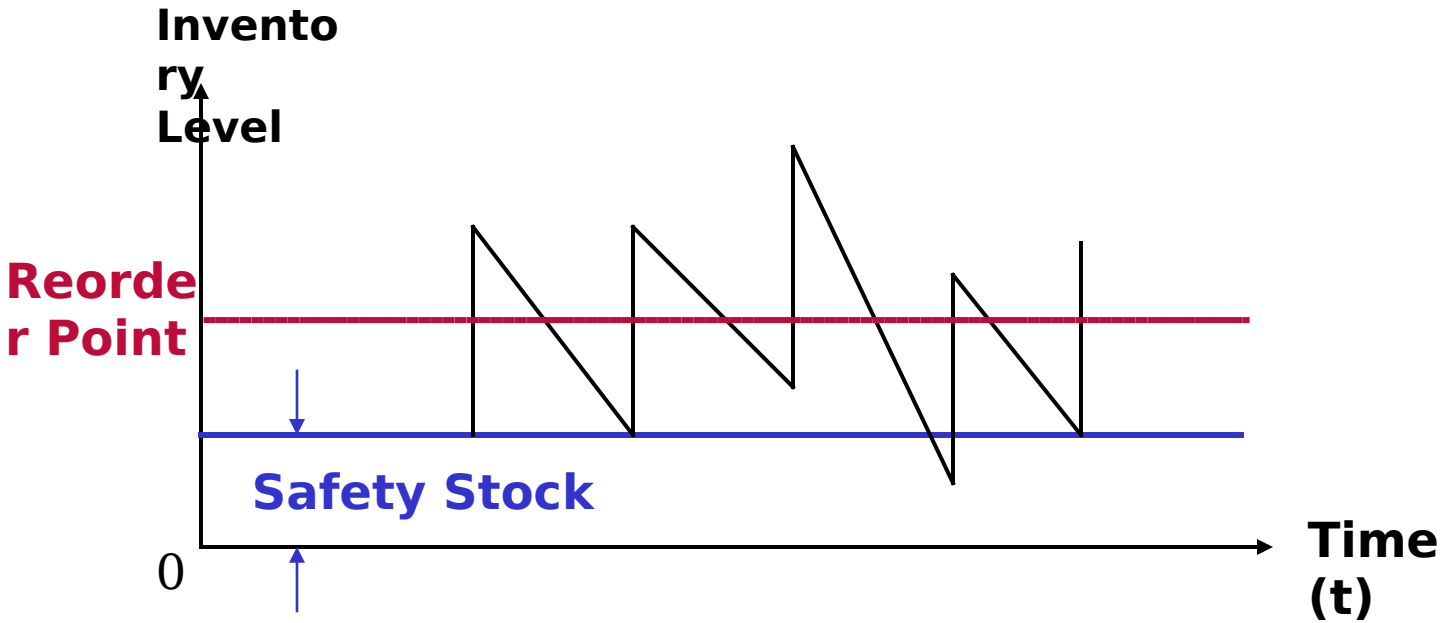
$$R = -\sqrt{\frac{2rc_3}{c_2} \cdot \frac{c_1}{c_1 + c_2}}$$

Reorder Point Quantity with replenishment lead time:

$$R = rt_2 - \sqrt{\frac{2rc_3}{c_2} \cdot \frac{c_1}{c_1 + c_2}}$$

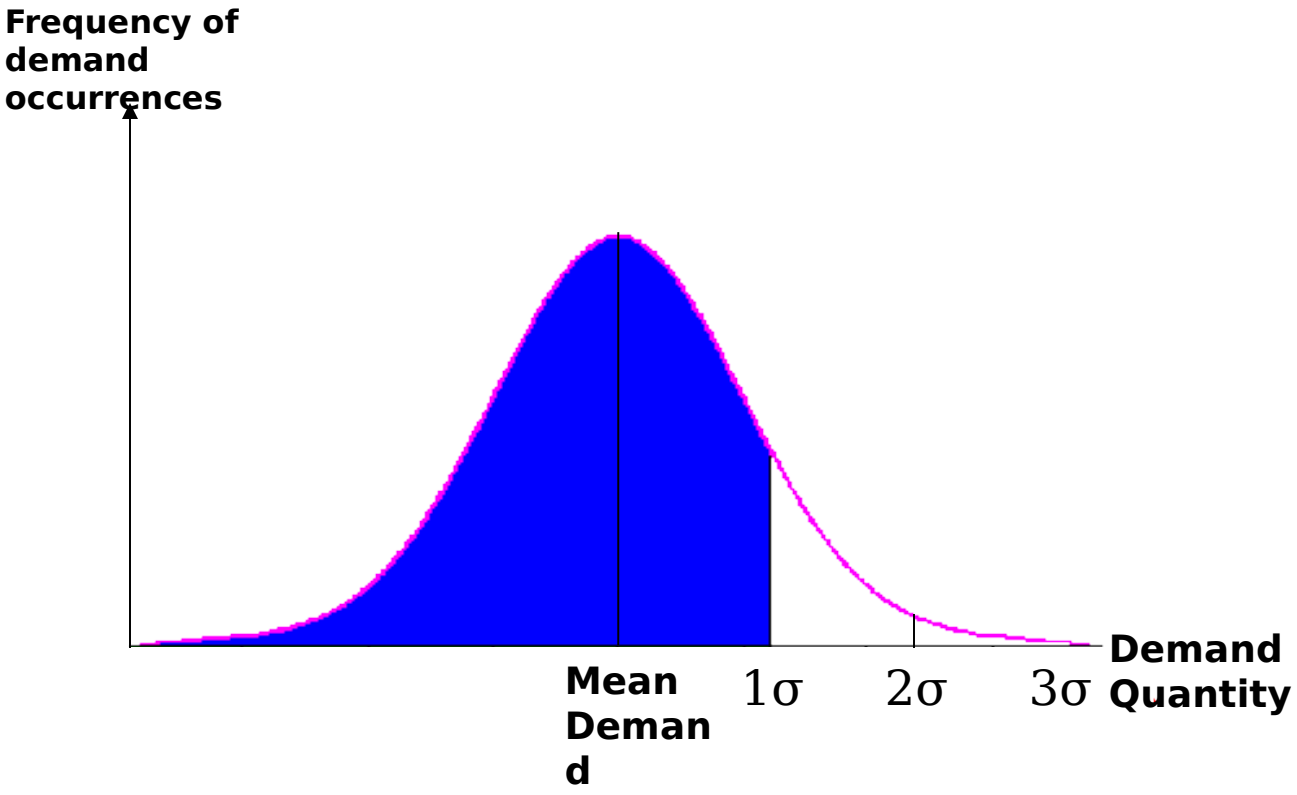
Safety Levels

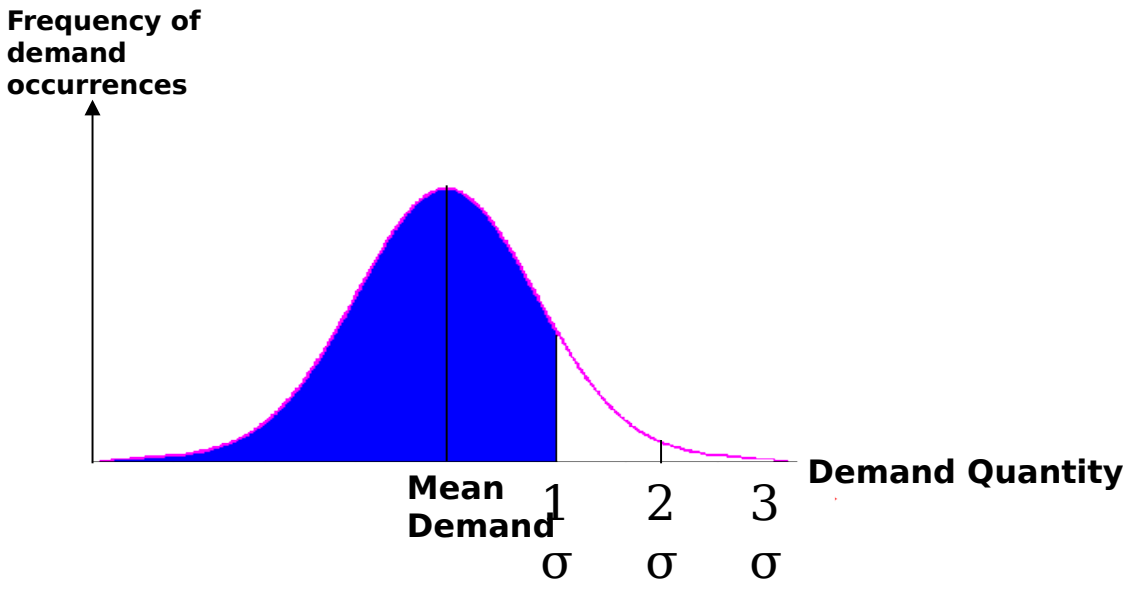
Safety stock is the extra quantity of stock carried as a protection against variable demand rates and a variable replenishment lead time as well as contingencies



Stocking for more than the average demand rate produces safety stock

Normal Distribution





Normal Distribution Properties:

- **The normal distribution is symmetrical about the mean**
- **The mean represent half (50%) the area under the curve**
- **The standard deviation is a measure of dispersion about the mean**
- **The mean plus 1 standard deviation (σ) represents approximately 84% of the area under the curve**

Usage of Normal Distribution to Determine Safety Level Stocks

- **Stocking for the mean demand is stocking to the 50% confidence level that the actual demand will not exceed mean demand over the specified time period**
- **Stocking for the mean demand plus 1 standard deviation (σ) is stocking to the 84% confidence level. Therefore, the actual demand should not exceed the mean demand +1 σ more than 16% of the time over the specified time period**
- **An order level equal to the mean demand plus X standard deviations is expected to prevent stock outs during Y% of the reorder periods**

X	Y
0.85σ	80%
1σ	84%
1.28σ	90%
1.65σ	95%
2σ	98%
2.32σ	99%
3σ	99.87%

Calculation of Mean & Standard Deviation

Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{n} x_i}{n}$

Standard Deviation $(\sigma) = \sqrt{\dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$

Example:

Reorder Period	Actual Demand	Error	Squared Error
i	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	220	30	900
2	170	-20	400
3	110	-80	6400
4	270	80	6400
5	210	20	400
6	160	-30	900

$\sum_{i=1}^{6} x_i = 1140$ $\sum_{i=1}^{6} (x_i - \bar{x})^2 = 15,400$

Mean Demand: $\bar{x} = \dfrac{1140}{6} = 190$ **Standard Deviation:** $\sigma = \sqrt{\dfrac{15,400}{5}} = \sqrt{3080} = 55.5$

Inventory Quantity Buildup

INVENTORY ELEMENTS	INV REQUIREMENT	ON ORDER QTY*ON-HAND QTY	IMPACTED BY LEAD-TIME
INSURANCE / RESERVE STOCK			X
NO			
SAFETY LEVEL STOCK		X	
YES			
	RECEIVE ORDER		
ADMINISTRATIVE LEAD TIME		X	
YES			
PRODUCTION LEAD TIME		X	YES
	REORDER POINT		
RE ORDER QUANTITY	ECONOMIC ORDER QUANTITY	X	
NO			
	REQUIREMENT OBJECTIVE		
UNFUNDED INSURANCE / RESERVES		X	
NO			
ECONOMIC RETENTION		X	
NO			
	MAX RETENTION LIMIT		
EXCESS TO DISPOSAL			

ABC Inventory Concept

A small number of items will account for most of the sales or cost dollars and therefore are the most important ones to control

Example Classification

Classification:	A	B	C
Items:	15%	35%	50%
Dollars:	65%	20%	15%

Classification of items by ABC method

ABC Classification of 14 products of a chemical company

Rank	Product Item Number	Montly Sales (000's)	Cumulative Percentages of Total Sales	Cumulative Percentages of Items	
1	D-204	5056	36.20%	7.10%	A
2	D-212	3424	60.70%	14.30%	
3	D-185-0	1052	68.20%	21.40%	B
4	D-191	893	74.90%	28.60%	
5	D-192	843	80.50%	35.70%	
6	D-193	727	86.00%	42.80%	
7	D-179-0	451	89.20%	50.00%	
8	D-195	412	92.20%	57.10%	C
9	D-196	214	93.50%	64.20%	
10	D-186-0	205	95.00%	71.50%	
11	D-198-0	188	96.50%	78.60%	
12	D-199	172	97.80%	85.70%	
13	D-200	170	99.00%	92.90%	
14	D-205	159	100%	100%	

The ABC classification is made by multiplying the annual usage of each product by its dollar value and then ranking these in descending order

ABC Inventory Management Concept

Expend minimal time & effort managing the low value "C" items

- **Carry plenty of low value items in stock**
- **Use minimal control & monitoring**

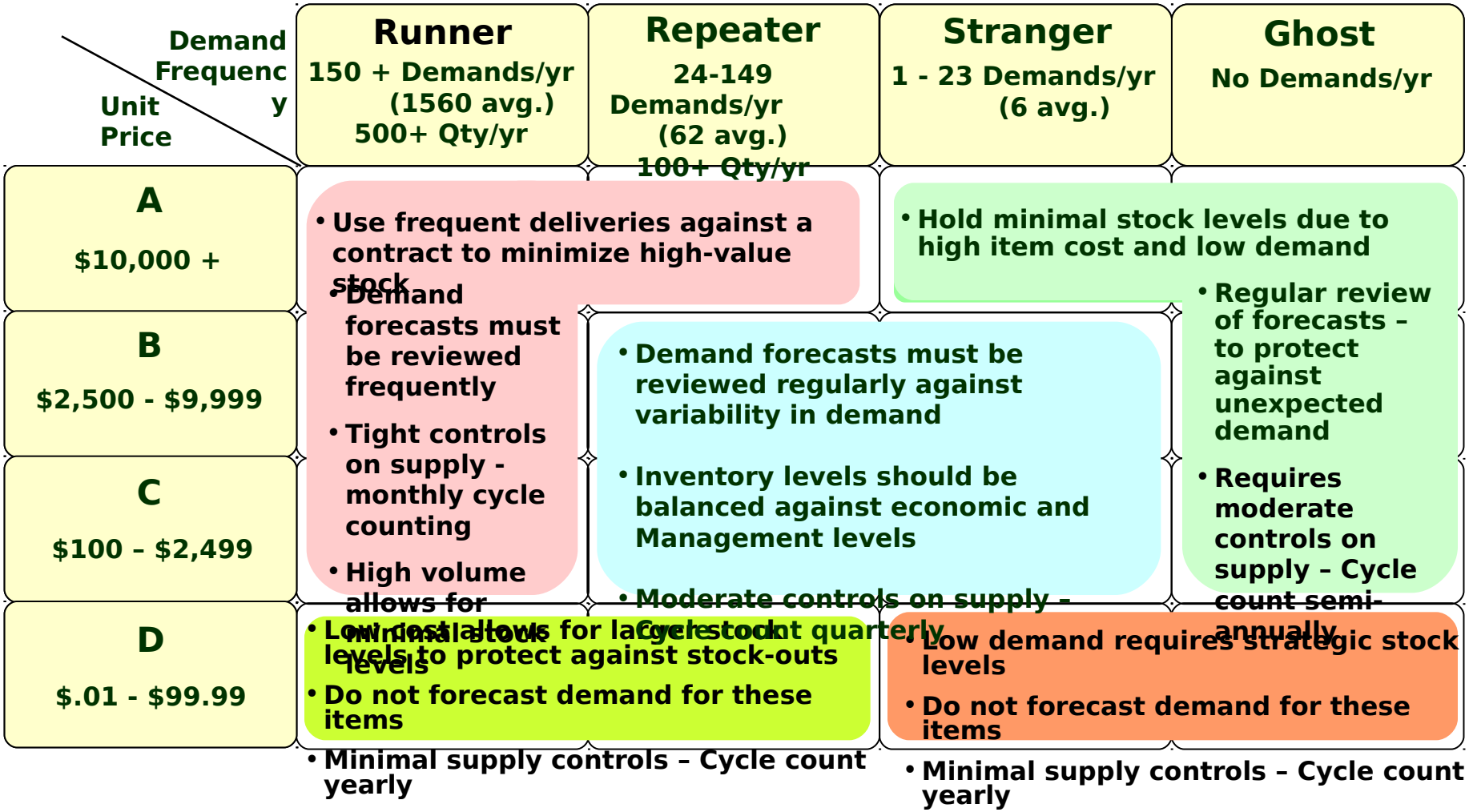
Apply maximum time & effort to closely control high value "A" items

- **Extra management decreases cost of high value items in stock**
- **Use maximum control & frequent reporting of inventory status**

Expend a medium amount of time & effort managing medium value "B" items

- **Medium management cost for medium value items in stock**

C-E LCMC Business Rule Guidelines


Demand Frequency
Unit Price
Runner
150 + Demands/yr
(1560 avg.)
500+ Qty/yr
Repeater
24-149
Demands/yr
(62 avg.)
100+ Qty/yr
Stranger
1 - 23 Demands/yr
(6 avg.)
Ghost
No Demands/yr
A
$10,000 +
B
$2,500 - $9,999
C
$100 – $2,499
D
$.01 - $99.99
• Use frequent deliveries against a contract to minimize high-value stock
• Demand forecasts must be reviewed frequently
• Tight controls on supply - monthly cycle counting
• High volume allows for minimal stock levels
• Demand forecasts must be reviewed regularly against variability in demand
• Inventory levels should be balanced against economic and Management levels
• Moderate controls on supply – Cycle count quarterly
• Low cost allows for larger stock levels to protect against stock-outs
• Do not forecast demand for these items
• Minimal supply controls – Cycle count yearly
• Hold minimal stock levels due to high item cost and low demand
• Regular review of forecasts – to protect against unexpected demand
• Requires moderate controls on supply – Cycle count semi-annually
• Low demand requires strategic stock levels
• Do not forecast demand for these items
• Minimal supply controls – Cycle count yearly